The Swinging Ball Problem

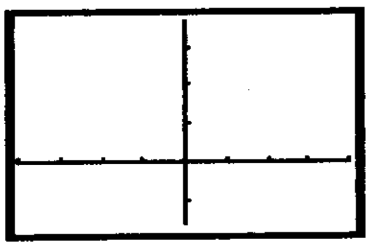
> **Introduction**: In this activity a ball is suspended by a piece of string above the floor. The ball swings back and forth as if it were a pendulum. You will find a mathematical model which represents the position of the ball with respect to time.

I. Collecting the Data

One student started the ball swinging back and forth as if it were a pendulum. Once the student was able to maintain a periodic motion and a consistent maximum height, the other class members took the following measurements:

- The height of the ball above the ground at its equilibrium position (when the string was vertical) was 0.33 ft.
- The height of the ball above the ground at its highest point was 1.7 ft.
- The horizontal distance from the equilibrium point to the highest position of the ball was 2.26 ft.
- The length of time for five complete swing cycles was 4.8 seconds. (A swing cycle is completed when the ball returns to the same position, for example the equilibrium position, and is headed in the same direction.)

On the axis below, sketch a graph of the motion of the ball while swinging. Label, as ordered pairs, the important points on your graph.



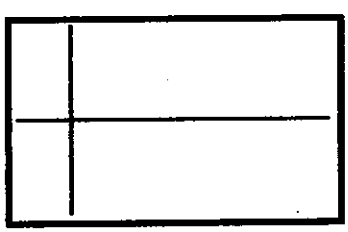
To obtain a model of the ball's motion, we need to divide this problem into two parts. First, we need to develop an equation for the horizontal position of the ball at any time, and, second, an equation for the vertical position of the ball at that time.

II. The Horizontal Position of the Ball at Any Time t: Finding a function for x(t).

Let's assume that at our start time, when $t=0$, the ball is at its equilibrium position. Therefore, its horizontal position at this time would also be zero. This means for our function $x(t)$, $x(0)=0$. Now we can find a few other points on our function pretty easily. Using the data collected and written above, how long will it take for the ball to complete one swing cycle? _______ This results in the point x(______) = _______. And, since the ball is also at the equilibrium position at half of one swing cycle, we have a third point of x(______) = _______.

Also, assume the ball begins its swing by moving to the right. How long would it take for the ball to reach its maximum horizontal distance from the equilibrium position? _________ This results in the point x(______) = _______. And finally, we can find a fifth point on our function when it reaches its largest distance in the negative direction from the equilibrium position. This results in the point x(______) = _______.

Sketch these points on the grid below and connect them to obtain your function $x(t)$. Be sure to label all of the important values.



From this graph we can obtain information about the Horizontal Position function x(t) and get its equation. Fill in the following from the graph of x(t).

Would it be easier to use a sine function or cosine function for the equation x(t)? __________

Explain:____

Amplitude: _______ Period: _________ Phase Shift: _________ Vertical Shift: __________

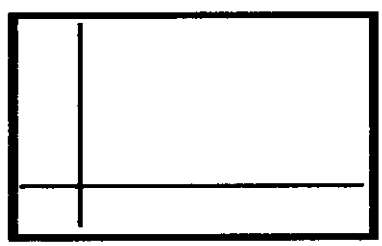
Equation for x(t): x(t) = ________________________________

To check your equation, graph it on your calculator. (Of course you have to enter y as a function of x, not x as a function of t.) Set up your window and see if it checks with your sketch above. If not, you need to find out what went wrong!

III. The Vertical Position of the Ball at Any Time t: Finding a function for y(t).

Again, we will assume that at our start time, when t=0, the ball is at its equilibrium position. Therefore, its vertical position at this time would be _______. This means for our function y(t), y(______) = _______. Now we need to find a few more points on our function. Using the data collected and written above, how long will it take for the ball to again be in this equilibrium position? (Hint: It is not one swing cycle! Think about how the ball is moving vertically!) This results in the point y(______) = _______.

When will the ball reach its maximum vertical height? This results in the point y(______) = _______. Sketch these three points on the grid below. Connect them to obtain one period of your function y(t). Be sure to label all of the important values.



From this graph we can obtain information about the Vertical Position function y(t) and get its equation. Fill in the following from the graph of y(t).

Would it be easier to use a sine function or cosine function for the equation y(t)? __________

Explain:____

Amplitude: _______ (Be careful!) Period: _________ (Not the same as x(t)) Phase Shift: _________ Vertical Shift: __________ Be careful!)

Equation for y(t): y(t) = ________________________________

Again, to check your equation, graph it on your calculator. (Of course you have to enter y as a function of x, not y as a function of t.) Set up your window and see if it checks with your sketch above. If not, you need to find out what went wrong!

IV. Putting It All Together

In order to see the motion of the ball, we need to put the two equations together. We are able to do this using Parametric Mode of our calculator. Put your calculator in Parametric Mode, then press the Y= key. Enter your Horizontal Position function x(t) in the X_{1T} row and your Vertical Position function y(t) in the Y_{1T} row. Press WINDOW and set Tmin=0 , Tmax=4.8 (for 5 complete swing cycles), and Tstep=0.05. Determine values for the rest of the WINDOW settings from your data and your first sketch. Press Graph to see the result. (You may want to press the ZOOM key and select 5:ZSquare to get a square window.)

To actually see the ball in motion change one other setting. Change the Style of your graph to the "ball" option. Watch the ball as it goes through the 5 swing cycles!